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Modeling and Simulation of Paper Folding

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Abstract

We study foldable objects by means of modeling, numerical simulation and geometrical analysis. For the latter, a relation between the fold angle, the geodesic and normal curvature of a crease curve is derived, giving rise to interesting geometrical observations of structures involving curved folds. To model objects including kinks, a 2D energy is identified as the Γ-limit of a 3D elastic energy that accounts for discontinuities of the deformation gradient along a prescribed curve. The reduced model is discretized by a discontinuous Galerkin method that allows for a practical description of foldable structures by neglecting gradient jumps of the deformation along the crease curve. A priori and a posteriori error estimates are derived for the corresponding linear model that describes configurations with small deflections and does not include the isometry condition. The latter provides a measure for the local accuracy of a finite element solution and helps guiding adaptive mesh refinement strategies. The discretized nonlinear model, that is based on a reconstruction of the Hessian and a relaxation of the isometry condition, Γ-convergences to the continuous model under appropriate density assumptions on smooth folded isometries. Various numerical experiments are carried out to study the physical behaviour of foldable objects like flytraps.