Georg Dolzmann, Universität Regensburg
Calculus of variations

In these three lectures, a short introduction into fundamental ideas and notions in the calculus of variations, in particular the direct method in the calculus of variations, is given. Illustrated with typical examples, difficulties concerning the existence of minima of functionals is discussed if the problem is posed on infinite dimensional spaces. The approach based on the passage from convergence in the norm to weak convergence together with suitable notions of convexity is presented. Some remarks concerning the idea of relaxation conclude the presentation.

Thorsten Bartel, TU Dortmund
Fundamentals of Structural Mechanics:

Structural mechanics" - distinguishing it from continuum mechanics - is the modeling of spatially three-dimensional solid continua by means of certain structures such as trusses, beams, plates as well as shells and the simulation of their mechanical behavior. These structures are fundamentally characterized by restrictive assumptions regarding the kinematics/deformations, which is simultaneously accompanied by a reduction of the spatial dimension of the underlying problem. On the one hand, structural mechanics is used for the calculation of analytical solutions for simple model systems (especially for comparison purposes). On the other hand, structural elements are also frequently used in finite element simulations to reduce the degrees of freedom used and thus the computation times.
The goal of this seminar is to understand the basic assumptions behind relevant theories ourselves and to derive the resulting equations on this basis. We thus go directly on the historical paths and in the footsteps of famous "mechanics" like Bernoulli, Kirchhoff & Co. The errors associated with the respective assumptions are always registered and first qualitatively estimated. Later, these are additionally analyzed and assessed quantitatively by comparison with finite element calculations. This also includes estimates of the range of application of the respective theory.
It is strongly recommended to bring a laptop or similar device on which the programming language Python including common libraries like numpy, scipy, matplotlib and an editor (e.g. Spyder, Jupyter) are installed. The Python codes needed to solve the problems covered in the seminar will be provided.

Manuel Friedrich, FAU Erlangen
Variational models for brittle fracture:

In the variational approach to brittle fracture, displacements and crack paths are determined from an energy minimization principle. The central objects are so-called

Griffith energies which comprise elastic contributions for the unfractured regions of the body and surface energies comparable to the size of the crack. As crack sets are not preassigned but a critical part of the minimization procedure, such issues are often referred to as a free-discontinuity problems. In this lecture series, we give an overview of classical and more recent existence results for Griffith functionals. In particular, we discuss the underlying framework of functions of bounded variation and deformation.

Klaus Hackl, Ruhr-Universität Bochum

Foundations of continuum mechanics for solids

We study the fundamental concepts of continuum mechanics of elastic solids at general, i.e. large, deformations. We start out looking at the underlying kinematics and introducing the various measures of deformation and strain, the Lagrangian and Eulerian time derivatives and their interrelations. We go on to derive the balance laws of the fundamental physical quantities as mass, energy, linear and angular momentum, which, as a consequence, lead to different concepts of stress. We have a look at the second law of thermodynamics and the principle of material frame indifference and their consequences for the formulation of constitutive laws. Finally, we give some examples for nonlinear elastic materials and look at some applications.

Sören Bartels, Albert-Ludwigs-Universität Freiburg

Numerical relaxation of nonconvex energy functionals

Nonconvex energy densities arise in certain simplified descriptions of phase transitions in crystalline solids. A drawback of the resulting mathematical models is the general ill-posedness, i.e., that minimizers of variational formulations may not exist. Relaxation procedures provide an attractive framework to obtain well-posed problems and to recover relevant information about material oscillations. In general, the required quasiconvex envelope is not accessible but efficient characterizations of upper and lower bounds are available. We discuss the reliable and efficient numerical approximation of polyconvex and rank-one convex envelopes. It turns out that the mathematically motivated polyconvex relaxation can be computed accurately with fast algorithms while the physically more meaningful ranke-one convex envelope leads to low convergence rates and high computational complexity.

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